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**BEFORE THE BOARD OF PATENT APPEALS  
AND INTERFERENCES**

Application Number: 10/783,645  
Filing Date: February 20, 2004  
Appellant(s): PLEASANT, DANIEL L.

**MAILED**

**DEC 12 2007**

**Technology Center 2100**

Calvin B. Ward (Reg. No. 30,896)  
For Appellant

**EXAMINER'S ANSWER**

This is in response to the appeal brief filed 11/08/2007 appealing from the Office action mailed 08/08/2007.

**(1) Real Party in Interest**

A statement identifying by name the real party in interest is contained in the brief.

**(2) Related Appeals and Interferences**

The examiner is not aware of any related appeals, interferences, or judicial proceedings, which will directly affect or be directly affected by or have a bearing on the Board's decision in the pending appeal.

**(3) Status of Claims**

The statement of the status of claims contained in the brief is correct.

**(4) Status of Amendments After Final**

The appellant's statement of the status of amendments after final rejection contained in the brief is correct.

**(5) Summary of Claimed Subject Matter**

The summary of claimed subject matter contained in the brief is correct.

**(6) Grounds of Rejection to be Reviewed on Appeal**

The appellant's statement of the grounds of rejection to be reviewed on appeal is correct.

**(7) Claims Appendix**

The copy of the appealed claims contained in the Appendix to the brief is correct.

**(8) Evidence Relied Upon**

*Jamneala et al.* (U.S. Patent No. 6,804,807), 10-12-2004

*Piratelli-Filho et al.*, (Uncertainty Evaluation in small angle Calibration using ISO GUM Approach and Monte Carlo Method, June 2003).

*Helisto et al.*, (Measurement Uncertainty in the 1/f noise region: Zener Voltage Standards, IEEE 2000).

**(9) Grounds of Rejection**

The following ground(s) of rejection are applicable to the appealed claims:

**Claims 1-4, and 6-16 are rejected under 35 U.S.C. 103(a)** as being unpatentable over Jamneala et al. (U.S. Patent No. 6,804,807), in view of Piratelli-Filho et al. (Uncertainty Evaluation in small angle Calibration using ISO GUM Approach and Monte Carlo Method, June 2003).

In considering the independent claim 1, Jamneala et al. substantially teaches a method of determining a measurement uncertainty of a test system comprising: developing a test system model having a plurality of uncertainty terms (*fig.5 (502), col.7 lines 63-64*); entering the test system model into a simulator (*fig.5 (504), col.7 lines 63-65*); running a sufficient number of iterations of the test system model on the simulator while randomly varying each of a first portion of the plurality of uncertainty terms within probability distributions to produce a statistically significant number of results of a selected parameter (*fig.5 (510-512), col.6 lines 51-58 & col.8 lines 12-23*); and evaluating the results to determine a measurement uncertainty of the selected parameter (*fig.5 (518), col.8 lines 12-23*). Although Jamneala et al. does not clear stat the term measurement uncertainty, he teaches simulating the system to obtain simulation results and match them with measured values (*see fig.5*). Nevertheless, Piratelli-Filho et al. substantially teaches a method for determining and evaluating measurement uncertainty using ISO GUM and Monte Carlo method (*see title*). Piratelli-Filho et al. and Jamneala et al. are analogous art because they are from the same field of endeavor and that the method teaches by Piratelli-Filho et al. is similar to that of Jamneala et al. Therefore, it would have been obvious to one ordinary skilled in

the art at the time of the applicant invention to combine the uncertainty evaluation method of Piratelli-Filho et al. with the method of Jamneala et al. because Piratelli-Filho et al. teaches obtaining expanded uncertainty results which proved simplified analysis (*see abstract*).

With regards to claim 2, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the simulator uses a harmonic balance simulation engine to produce the results (*see Jamneala et al. col.6 lines 7-17 (ADS simulator); also see Piratelli-Filho et al. section 2.2-3*).

As per claims 3, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the simulator uses a time-domain simulation engine to produce the results (*see Jamneala et al. col.6 lines 7-17 (ADS simulator); also see Piratelli-Filho et al. section 2.2-3*).

With regards to claim 4, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the simulator uses a linear S-parameter simulation engine to produce the results (*see Jamneala et al. col.6 lines 7-17 (ADS simulator); also see Piratelli-Filho et al. section 2.2-3*).

Regarding claim 6, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the plurality of uncertainty terms includes a test instrument uncertainty term for a test instrument in the test system (*see Jamneala et al. fig.1; also see Piratelli-Filho et al. pg.1-4*).

As per claim 7, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the test instrument uncertainty term is selected from the group consisting

of a temperature drift uncertainty term, an aging drift uncertainty term, an accuracy uncertainty term, and a repeatability uncertainty term (*see Piratelli-Filho et al. pg.1-4*).

Regarding claim 8, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the test system model includes a device under test and the step of running the sufficient number of iterations provides a first frequency to the device under test, and the results of the selected parameter are at a second frequency (*see Jamneala et al. fig.2B-4, col.1 lines 60-64& col.6 line 41-col.8 line 23; also see Piratelli-Filho et al. pg.1-4*).

With regards to claim 9, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the second frequency is a harmonic of the first frequency (*see Jamneala et al. fig.2B-4, col.6 line 41-col.7 line 38; also see Piratelli-Filho et al. pg.1-4*).

As per claim 10, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the second frequency is a mixing product of the first frequency and a third frequency (*see Jamneala et al. fig.2B-4, col.6 line 41-col.7 line 38; also see Piratelli-Filho et al. pg.1-4*).

Regarding claim 11, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the test system model includes a test instrument as a device under test (*see Jamneala et al. fig.1, col.1 lines 60-64, col.3 line 50-col.4 line 6; also see Piratelli-Filho et al. pg.1-4*).

As per claim 12, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the test system model includes a test fixture comprising a plurality of

switches and a plurality of cables (*see Jamneala et al. fig.1, col.1 lines 60-64, col.3 line 50-col.4 line 6; also see Piratelli-Filho et al. pg.1-4*).

With regards to claim 13, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach the step of running occurs at a first operating condition and further comprising steps of: running a sufficient number of iterations of the test system model on the simulator at a second operating condition while randomly varying each of the first portion of the plurality of uncertainty terms within probability distributions to produce a statistically significant number of second results of the selected parameter (*see Jamneala et al. fig.2B-5, col.6 line 41-col.8 line 23; also see Piratelli-Filho et al. pg.1-4*); and evaluating the second results to determine a second measurement uncertainty of the selected parameter (*see Jamneala et al. fig.2B-5, col.6 line 41-col.8 line 23; also see Piratelli-Filho et al. pg.1-4*).

As per claim 14, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach the step of running is done using a first type of simulation engine and further comprising steps of: running a second sufficient number of iterations of the test system model on the simulator using a second type of simulation engine while randomly varying each of the first portion of the plurality of uncertainty terms within probability distributions to produce a statistically significant number of second results of a second selected parameter (*see Jamneala et al. fig.2B-5, col.6 line 41-col.8 line 23; also see Piratelli-Filho et al. pg.1-4*); and evaluating the second results to determine a second measurement uncertainty of the second selected parameter (*see Jamneala et al. fig.2B-5, col.6 line 41-col.8 line 23; also see Piratelli-Filho et al. pg.1-4*).

Regarding claim 15, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach the step of developing a computer-readable library of test system components with uncertainty terms, and wherein the step of entering the test system model into the simulator includes loading uncertainty terms associated with the test system components from the computer-readable library (*col.8 lines 35-45*).

As per claim 16, the combined teachings of Jamneala et al. and Piratelli-Filho et al. substantially teach that the step of developing the test system model includes automatically generating system specifications (*fig.5, col.8 lines 12-23; also see Piratelli-Filho et al. pg.1-4*).

**Claim 5 is rejected under 35 U.S.C. 103(a)** as being unpatentable over Jamneala et al. (U.S. Patent No. 6,804,807), in view of Piratelli-Filho et al. (Uncertainty Evaluation in small angle Calibration using ISO GUM Approach and Monte Carlo Method, June 2003), and further in view of Helisto et al. (Measurement Uncertainty in the 1/f noise region: Zener Voltage Standards, IEEE 2000).

Regarding claim 5, Jamneala et al., as modified by Piratelli-Filho et al. and applied to claims 1-4, and 6-16 above, teaches most of the instant invention. However, they do not clearly teach that the plurality of uncertainty terms includes a noise term. Helisto et al. substantially teaches a Measurement Uncertainty in the 1/f noise region (*see title, pg.401-402*). Piratelli-Filho et al., Jamneala et al., and Helisto et al. are analogous art because they are from the same field of endeavor and that the method teaches by Helisto et al. is similar to that of Jamneala et al. and Piratelli-Filho et al. Therefore, it would have been obvious to one ordinary skilled in the art at the time of the applicant invention to combine the uncertainty measurement method of Helisto et



al. with the method of Jamneala et al. and the uncertainty evaluation method of Piratelli-Filho et al. because Helisto et al. teaches a development that enable the measurements down to the fundamental noise limit of metrological devices (*see pg.402*).

## 10) Response to Arguments

Appellant argues: **(Claim 1)**

*"There is no teaching of a probability distribution described the statistical variability of the determined inductance"*, the Examiner respectfully notes that claim does not even mention a determined inductance, much less a probability distribution of a determined inductance, as argued by the Appellant, although the claims does show a probability distribution. Nevertheless, Jamneala et al., used as a primary reference, teaches an arbitrary distribution of currents and further teaches making assumption regarding a plurality of inductance values used during calibration of a model (*see col.4 lines 19-48 and col.6 lines 7-61*), the Examiner reasserts that the claim, as presented, does not show *"a probability distribution described the statistical variability of a determined inductance"*, as argued by the Appellant. The Examiner respectfully notes that it is the combination of both Jamneala et al. and Piratelli-Filho et al. that is relied upon in rejecting the instant claim, as Piratelli-Filho et al. further does teach generating variability in accordance a probability distribution of a number of parameters (*see page.1, and the ISO GUM and the Monte Carlo techniques*).

Regarding Applicant assertion that *"the Examiner has failed to make a prima facie case for obviousness with respect to claim 1 and the claims dependent therefrom"* is acknowledged.

However, the Examiner notes that the ground of rejection above clearly shows a complete

mapping of the cited prior art to the instant claims addressing all claims limitations. The Examiner, in the below ground of rejection, points to specific portion of the references for reasons/motivation to combine the references and that a prima facie has clearly been established. Jamneala et al. and Piratelli-Filho et al are analogous art because they are from the same field of endeavor, and therefore the Examiner has properly rejection the claims in accordance with the practices and procedures set forth in the MPEP.

Appellant argues: **(Claims 2-4)**

Regarding Appellant's assertion that *"an ADS includes a harmonic balance engine, and may also include time-domain or linear S-parameter simulation"* is acknowledged. However, appellant's argues that "the Examiner has not pointed to any such teaching in Jamneala or Piratelli-Filho that any of the three types of simulation engine is utilized in the simulation", the Examiner respectfully disagrees and asserts that Jamneala et al. does disclose an ADS simulator that inherently includes the simulation engines of the instant claims (*col.6 line 7-61 and fig.5*), and one ordinary skilled in the art would clearly understand that the ADS Simulation, as taught by Jamneala et al., substantially includes harmonic balance simulation, the time-domain simulation, and the S-parameter simulation engines and produce the simulation results. The Examiner did mention the ADS non-patent document made of record merely for support of the Examiner assertion of the ADS simulator discloses by Jamneala et al. Nevertheless, Piratelli-Filho et al., used as secondary reference, further teaches the use of the well-known Monte Carlo simulator and further produces the simulation results claimed by the applicant (*see pg. 2-3*).

Appellant argues: **(Claims 8-10)**

*“The Examiner has not pointed to any specific teachings in either of the cited prior art references that discloses probing the DUT at a first frequency and measuring the results at a second frequency”, the Examiner respectfully disagrees and notes that Jamneala et al. does teach running a number of simulation iteration on the device to obtain simulation best-fit results (see for example col.6 lines 7-61 and col.7 line 61-col.8 line 23), and that the simulation of Jamneala et al. is performed using wide range of different frequencies to produces these simulation results (see simulation results of fig.2B-4).*

Appellant argues: **(Claim 12)**

*“There is no teachings in the cited figures and passages in Jamneala of any switch or cable being included in the test system model”, the Examiner respectfully notes that although it the combination of Jamneala et al. and Piratelli-Filho that is relied upon in the rejection of the claims, Jamneala et al.'s system does includes transmission lines/cables and that during calibration of the test system, Jamneala et al. does teach removes static error arising from the test cable and that elements 24,34,26 of fig.1 are substantially switches with element 22 being a transmission line/cable for example (see also col.3 lines 29-47 and col.3 line 51-col.4 line 6).*

Appellant argues: **(Claim 13)**

*“There is no uncertainty terms in the system of taught by Jamneala, and that no measurement uncertainties are determined by evaluation of a set of second results”, the Examiner disagrees and notes that Jamneala et al. does teach making a number of assumption which are substantially uncertainty terms to includes inductance, and further teaches an arbitrary distribution of currents and inductance assumption during a conventional calibration of the model (see col.4 lines 19-col.5 line 62), and further teaches obtaining simulation results using a*

variation of measured values for inductance and mutual inductance to obtain a best-fit results using iterative simulation (*col.6 lines 7-61 and col.7 line 61-col.8 line 23*). The Examiner further notes that it the combination of Jamneala et al. and Piratelli-Filho that is relied upon for support in rejecting the instant claim, as Piratelli-Filho et al. is fully directed to an uncertainty evaluation method using known simulation techniques(*see page.1*), including measuring uncertainty considering a variety of uncertainty terms , including a number of operating condition such as the difference of temperature, variation of length, standard deviation, etc., and further teaches generating random numbers in accordance with expected probability distribution and specified range of variation (*see the Monte Carlo and ISO GUM approach pg.2-3*).

Appellant argues: **(Claim 14)**

*“The Examiner has not pointed to any such teaching of the simulation engines in claim 14”*, the Examiner respectfully notes the ADS simulator teaches by Jamneala et al. inherently includes the multiple simulation engines of the instant claims (*col.6 line 7-61 and fig.5*), and one ordinary skilled in the art would clearly understand that the ADS Simulation, as taught by Jamneala et al., substantially includes harmonic balance simulation, the time-domain simulation, and the S-parameter simulation engines and produce the simulation results, and that Jamneala et al. further teaches obtaining simulation results using a variation of measured values for inductance and mutual inductance to obtain a best-fit results using iterative simulation (*col.6 lines 7-61 and col.7 line 61-col.8 line 23*). The Examiner further notes that it the combination of Jamneala et al. and Piratelli-Filho that is relied upon for support in rejecting the instant claim, as Piratelli-Filho et al. is fully directed to an uncertainty evaluation method using known simulation techniques(*see page.1*), including measuring uncertainty considering a variety of uncertainty

terms and generating random numbers in accordance with expected probability distribution and specified range of variation (*see the Monte Carlo and ISO GUM approach pg.2-3*). The Examiner did mention the ADS non-patent document made of record merely for further support of the Examiner assertion of the ADS simulator discloses by Jamneala et al. Nevertheless, Piratelli-Filho et al., used as secondary reference, further teaches the use of the well-known Monte Carlo simulator and further produces the simulation results (*see the Monte Carlo and ISO GUM approach pg.2-3*).

Appellant argues: **(Claim 15)**

“There is no teaching in the passage regarding loading of the uncertainty term”, the Examiner disagrees and notes that Jamneala et al. does teach loading a model into the simulator and making a number of assumptions with regards to uncertainty terms to include inductance, and further teaches an arbitrary distribution of currents during a conventional calibration of the model (*see col.4 lines 19-col.5 line 62*), and run simulation iteratively to obtain simulation results using a variation of measured values for inductance and mutual inductance and obtain a best-fit results using the iterative simulation (*col.6 lines 7-61 and col.7 line 61-col.8 line 23*). The Examiner further notes that it the combination of Jamneala et al. and Piratelli-Filho that is relied upon for support in rejecting the instant claim, as Piratelli-Filho et al. is fully directed to an uncertainty evaluation method using known simulation techniques(*see page.1*), including measuring uncertainty considering a variety of uncertainty terms , and further teaches generating random numbers in accordance with expected probability distribution and specified range of variation (*see the Monte Carlo and ISO GUM approach pg.2-3*).

Appellant argues: **(Claim 16)**

*"The simulation of Jamneala et al. does not disclose the automatic generation of system specification",* the Examiner respectfully disagrees and asserts that it is the combination of Jamneala et al. and Piratelli-Filho et al. that is relied upon for support in rejecting the instant claim. Jamneala et al., used as the primary reference, fig.5 show the load of the model into the simulator, the GSG model is created having through path, inductance, etc. (see also col.7 line 61-col.8 line 45), and Piratelli-Filho, used as a secondary, is fully directed to an uncertainty evaluation method using known simulation techniques(see page.1), including measuring uncertainty considering a variety of uncertainty terms and automatically generating system specification, in accordance with expected probability distribution and specified range of variation (see the Monte Carlo and ISO GUM approach pg.2-3). Upon simulation of the system both Jamneala et al. and Piratelli-Filho, used known simulation techniques to produce the simulation results (see Jamneala et al. fig.2B-5, col.6 lines 7-61 and col.7 line 61-col.8 line 45; also see Piratelli-Filho et al. pg.2-3).

Appellant argues: **Claim 5**

Regarding Applicant assertion that *"the Examiner has failed to make a prima facie case for obviousness with respect to claim 5"* is acknowledged. However, the Examiner notes that the ground of rejection above clearly shows a complete mapping of the cited prior art to the instant claims addressing all claims limitations. The Examiner, in the above ground of rejection; points to specific portion of the references for reasons/motivation to combine the references and that a prima facie has clearly been established. Jamneala et al., Piratelli-Filho et al., and Helisto et al. are analogous art because they are from the same field of endeavor, and therefore

the Examiner has properly rejection the claims in accordance with the practices and procedures set forth in the MPEP.

Having addressed all of the Appellant's arguments, the Examiner further asserts that the grounds of rejection above along with the response to arguments fully support the Examiner's position in the rejection of the instant claims and that the rejection should be maintained. The Examiner respectfully notes that both Jamneala et al. and Piratelli-Filho references should be considered entirely in supporting the Examiner's position in the rejection of the instant claims.

**(11) Related Proceeding(s) Appendix**

No decision rendered by a court or the Board is identified by the examiner in the Related Appeals and Interferences section of this examiner's answer.

For the above reasons, it is believed that the rejections should be sustained.

Respectfully submitted,

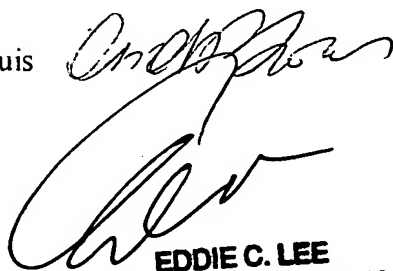
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Measurement uncertainty in the  $1/f$  noise region: Zener voltage standards

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## Abstract

The measurement uncertainty is studied in terms of the Allan variance when the uncertainty is limited by  $1/f$  noise. The deadtime in measurements determines the time scale of best measurement uncertainty. About ten equally spaced samples seem to map the noise spectrum well enough and no uncertainty improvement results from further averaging. The voltage difference of two Zener standards in the frequency band  $10^{-6}$  Hz –  $10^3$  Hz was studied and closely  $1/f$  behaviour was detected. The noise floor of one Zener is about 70 nV.

## Introduction

Theoretically, the noise in many measurements can be described as  $1/f$  noise limited by white noise at high frequencies. It is often understood that the  $1/f$  corner frequency  $f_0$  corresponds to an averaging time after which the measurement result does not improve. On the other hand, in metrology it is common to average hundreds of single measurements independent of the noise properties of the device under test. Also, often the standard deviation of the mean is used as a measure of uncertainty without detailed knowledge of the noise properties.

Properties of signal fluctuations are commonly described by their power spectral density (PSD). In frequency metrology, another quantity – the Allan variance is often used [1]. In a way, the Allan variance estimates the future uncertainty of result of the measurement based on its present value.

The long term ( $> 1$  month) behaviour of Zener voltage standards is studied in this work both in terms of the PSD and the Allan variance. The noise PSD of Zener references has been studied earlier in e.g. Refs. [2] and [3].

## Theory

We assume that the device is described by the spectral density function  $S(f) = S_0(1 + f_0/f)$ . In practice, the results are corrupted by digital sampling and filtering. Severe aliasing and leakage may be introduced especially when a DVM is the measuring instrument.

Let us consider a measurement with deadtime shown in the insert of Fig. 1, where  $\tau$  is the integration time

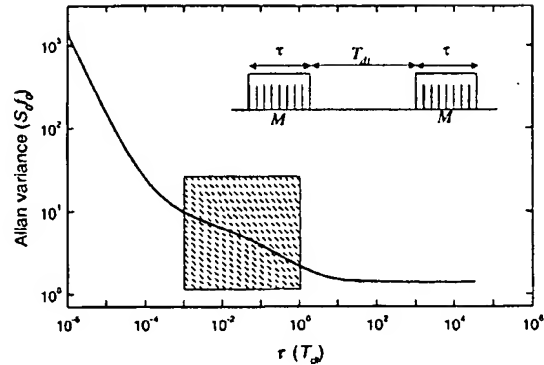


Fig. 1. Allan variance with deadtime.  $T_{dt} = 1000/(4 \ln 2 \cdot f_0)$ . Insert: measurement procedure. Region 2 is shadowed (see text).

and  $\tau + T_{dt}$  is the total duration of one measurement. The corresponding Allan variance can be calculated from  $S(f)$ : [4]

$$\sigma_y^2(2, \tau; T_{dt}) = \frac{2}{\pi \tau} \int_0^\infty S\left(\frac{2u}{\tau}\right) \frac{\sin^2 u}{u^2} \sin \left[ \left(1 + \frac{T_{dt}}{\tau}\right) u \right] du. \quad (1)$$

Typically this could correspond to a measurement with a DVM ( $T_{dt} \sim 1$  s) or to an international comparison ( $T_{dt} \sim 1 - 7$  d).

A curve calculated from Eq. 1 is shown in Fig. 1. The behaviour of  $\sigma_y^2$  is divided into three regions:

1. white noise region  $\tau < (4 \ln 2 \cdot f_0)^{-1}$ , where the variance diverges as  $1/\tau$ ,
2. deadtime limited region  $(4 \ln 2 \cdot f_0)^{-1} < \tau < T_{dt}$ , where logarithmic behaviour is seen, and
3.  $1/f$  noise region  $\tau > T_{dt}$ , where the Allan variance becomes constant.

In other words: to minimize the measurement uncertainty, one should average at least as long as the deadtime in the measurement, if  $T_{dt} > (4 \ln 2 \cdot f_0)^{-1}$ . However, after  $\tau \approx 3 \cdot T_{dt}$ , not much can be gained by further averaging.

## Measurements and results

To test the  $1/f$  noise hypothesis, the voltage difference between the 10 V outputs of two Fluke 732B Zener voltage references was measured with a Keithley 182



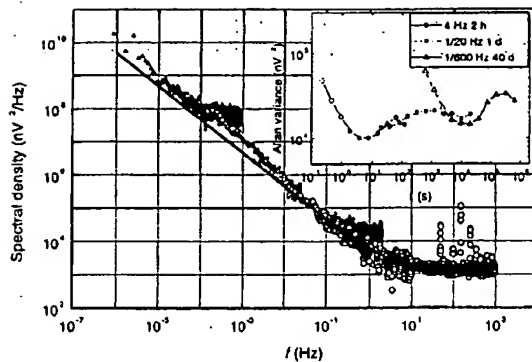


Fig. 2. Spectral density and Allan variance of 10-V Zener voltage fluctuations.

nanovoltmeter. No filtering was used except the setting NPLC 1. Three sets of measurements were obtained, in the first the sampling rate was 4 Hz and duration 2 h, in the second 1/20 Hz and 1 day and in the third 1/600 Hz and 40 days, correspondingly. Linear drift was subtracted from the data before the analysis. In addition, the 'high' frequency spectrum was measured with a spectrum analyzer and a low noise preamplifier.

The power spectral density and Allan variance of Zener voltage fluctuations are shown in Fig. 2. Corner frequency  $f_0 = 3.5$  Hz and spectral density  $S_0^{1/2} = 36.7$  nV/ $\sqrt{\text{Hz}}$  were obtained from the spectrum analyser data (open circles). Only a small deviation from this  $1/f$  behaviour (solid curve) is observed even at the smallest frequencies and it can be attributed to environmental variations and sampling effects (compare the insert of Fig 2).

In the insert of Fig. 2 the Allan variance of the data is plotted. The three measurements with different sampling times fit again well together excluding the first points of the 40 d data. This is due to 'aliasing' of  $1/f$  noise because of the large deadtime present. There seems to be a shallow minimum at 1-10 s intervals, but again the first points of the 4 Hz curve are dominated by deadtime. At the largest time intervals, the variance slightly increases, which may indicate variations in the laboratory conditions at  $\geq 1$ -d time scales.

In Fig. 3, the variance of the data of the second measurement set is shown, when  $M$  equally spaced data points within interval  $\tau$  are averaged and the variance of such subsequent averages is plotted as a function of  $M$  for different values of  $\tau$ . Now, in principle  $T_{dt} \approx 0$ , but there is substantial, increasing deadtime between individual nonaveraged data points as  $\tau$  increases (see

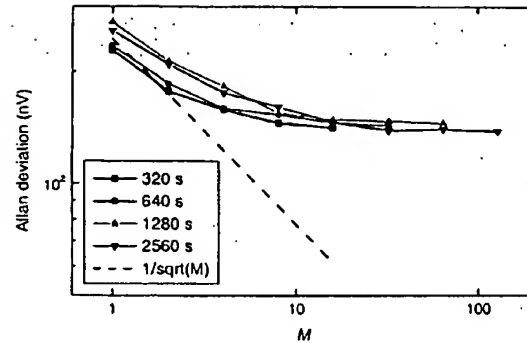


Fig. 3. Allan deviation as a function of number of averages within different time separations  $\tau$  (measurement 1/20 Hz, 1 d). Dashed line shows the  $1/\sqrt{M}$  behaviour of white noise.

the insert in Fig. 1). The curve shapes are almost identical: first the variance decreases but the limiting Allan variance is obtained when  $M \approx 10$ . Evidently, the variance of the mean, which assumes a  $1/M$  decrease, would lead to a considerably underestimated type A uncertainty.

## Conclusions

Development of present day instrumentation enables measurements down to the fundamental noise limit of metrological devices. In the  $1/f$  noise region, new types of measurement practices are needed. It appears useless to average more than about 10 samples. In, e.g., international comparisons, the deadtime spent on transportation and instrument stabilization should be minimized and the measurement time need not be more than three times this deadtime. Repeated fast comparisons yield smaller uncertainty than one 'slow' comparison.

Compared to Ref. [3], we observe very closely  $1/f$  behaviour at low frequencies. Measurement of a Zener voltage standard seems to be almost invariably dominated by its  $1/f$  noise. With a typical 10-V Zener standard, the attainable accuracy in, e.g. voltage comparisons is at best 70 nV. We intend to continue the study to even longer measurement periods to see the effect of environmental variations on the observed noise.

## REFERENCES

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## UNCERTAINTY EVALUATION IN SMALL ANGLE CALIBRATION USING ISO GUM APPROACH AND MONTE CARLO METHOD

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**Abstract** - This work investigates the determination of measurement uncertainty in calibration of small angle measurement instruments. After attentive study of the sources of variation, an expression was developed to determine the measurement uncertainty. Two approaches were used to determine the measurement uncertainty: ISO GUM and Monte Carlo simulation method. An example is presented and the calibration of an Electronic level was carried out using a sine table at the Metrology Laboratory, in the University of Brasilia, Brazil. The expanded uncertainty results showed good agreement of both techniques and Monte Carlo method proved to simplify analysis when uncertainty involves expressions with some degree of complexity.

Key words: uncertainty, Monte Carlo method, small angle

### 1. INTRODUCTION

Nowadays, the search for quality has been promoting an increasingly effort in the enterprises that are looking for ISO 9000 standard certification. As a consequence, the demand for calibration services has been growing up. One of the requirements to attain ISO 9000 certification is that calibration results must be expressed in conjunction with the measurement uncertainty.

The measurement uncertainty determination is addressed by an ISO publication since 1993 whose well known title is Guide to the Expression of Uncertainty in Measurement (GUM) [1]. This document is largely used all around the world and it was translated to several different languages. Some difficulties related to its application comes from that cases where complex formulae relating input and output quantities is developed. A revision of this standard is taking place in order to address this and other key aspects of the GUM.

The calibration of small angle measurement instruments like the Spirit level and the Electronic level used in mechanical industry is a necessary effort to assure quality of the measurements [2]. This may be accomplished using a sine bar and gauge blocks to establish standard angles and than comparing to the measured angles. The Measurement uncertainty may be determined according to ISO GUM [1] but it may be pointed out that trigonometric relationship among variables brings some difficult when deriving the expression to obtain the measurement uncertainty formulae.

An approach that is growing in acceptance by researchers is the Monte Carlo simulation and its application is performed generating the variability according to expected probability distributions of each variable [3, 4]. In this work the calibration of an Electronic Level is carried out to compare these two approaches used to determine the measurement uncertainty.

### 2. CALIBRATION AND UNCERTAINTY

Since the plane angle is defined in terms of the full circle and there is no primary standard artifact for the angle, angle measurement is better performed when the round angle is divided as equal as possible. Thus, the calibration of angle measurement instruments may be carried out by using a measuring table or by using a sine bar. The option by the sine table may be done when dealing with calibration of small angle measuring instruments. In this case, the angle determined is related to the length of the gauge blocks and its uncertainty is closely related to the gauge blocks uncertainties.

An experimental assembly scheme to measure an inclination  $\theta$  using an Electronic Level placed over a sine table is showed in Fig. (1). If this Electronic Level has a bias, the systematic error may be determined comparing standard angle  $\theta_s$  obtained on sine table with angle  $\theta$  measured using the Electronic Level. The calibration procedure involves gradually increasing the standard angle  $\theta_s$  and recording the correspondent indication  $\theta$  to draw a calibration curve.

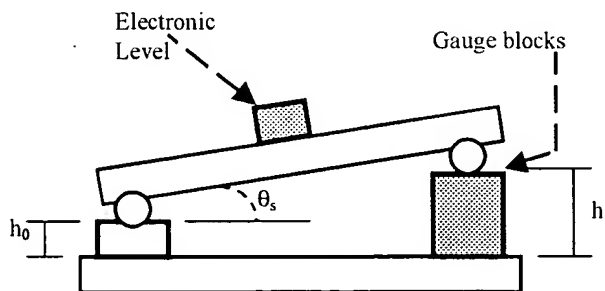


Figure 1 – Experimental assembly at sine bar

## 2.1 – ISO GUM approach

According to ISO Guide [1], determination of measurement uncertainty begins setting out a mathematical model that holds all variables influencing the measurand. The first step is the investigation of the variables involved in the measurement using the Electronic Level. It was considered that measured angle  $\theta$  depends on standard angle at sine bar ( $\theta_s$ ), instrument bias ( $\Delta\theta_s$ ), bias associated to instrument resolution ( $\Delta R$ ), bias associated to temperature variation in relation to reference temperature 20° C ( $\Delta T_{20}$ ), bias associated to temperature difference between instrument and gauge used ( $\Delta T_{diff}$ ) and roundness of sine bar cylinder ( $\Delta C$ ). A mathematical model that represents the effect of these variables is showed in Eq. (1). The effect of drift was not considered.

$$\theta = \theta_s + \Delta\theta_s + \Delta R + \Delta T_{20} + \Delta T_{diff} + \Delta C \quad (1)$$

The measurement uncertainty model is obtained applying error propagation on Eq. (1) and thus we have Eq. (2). As showed, the combined standard uncertainty ( $u_\theta$ ) of measured angle is a function of the standard uncertainties associated to the variability of measured angles ( $u_{\Delta\theta_s}$ ), the standard angle established using the sine bar ( $u_{\theta_s}$ ), the instrument resolution ( $u_{\Delta R}$ ), the temperature variation in relation to standard reference ( $u_{\Delta T_{20}}$ ), the temperature difference between standard and the instrument ( $u_{\Delta T_{diff}}$ ) and the roundness of sine bar cylinder ( $u_{\Delta C}$ ).

$$u_\theta^2 = u_{\theta_s}^2 + u_{\Delta\theta_s}^2 + u_{\Delta R}^2 + u_{\Delta T_{20}}^2 + u_{\Delta T_{diff}}^2 + u_{\Delta C}^2 \quad (2)$$

The standard angle depends on trigonometric relation established when gauge blocks are positioned over the sine table to perform measurement. Thereby the standard angle  $\theta_s$  is calculated by arcsine function of the ratio between the height of gauge blocks ( $h_1$ ) decreasing the initial height on sine table ( $h_0$ ) and the distance between cylinders of sine table ( $L$ ). Eq. (3) shows this expression.

$$\theta_s = \arcsin\left(\frac{h_1 - h_0}{L}\right) \quad (3)$$

Since the standard angle  $\theta_s$  determined by Eq. (3) is a function of the height of gauge blocks, the uncertainty  $u_{\theta_s}$  is related to the gauge blocks standard uncertainties  $u_{h_0}$  and  $u_{h_1}$  and the sine bar length standard uncertainty  $u_L$ . After the application of error propagation, the standard uncertainty  $u_{\theta_s}$  is determined according to Eq. (4).

$$u_{\theta_s}^2 = \left(\frac{\partial \theta}{\partial h_0}\right)^2 \cdot u_{h_0}^2 + \left(\frac{\partial \theta}{\partial h_1}\right)^2 \cdot u_{h_1}^2 + \left(\frac{\partial \theta}{\partial L}\right)^2 \cdot u_L^2 \quad (4)$$

The sensitivity coefficients related to the first, second and third terms in Eq.(4) are determined by partial derivatives of angle  $\theta$  in respect to  $L$ ,  $h_0$  and  $h_1$  respectively. Since there are two parameters in these expressions, it must be considered the variable transformation  $u = (h_1 - h_0) / L$ .

Thus, these coefficients are calculated by Eq. (5), (6) and (7).

$$a_1 = \frac{\partial \theta}{\partial h_0} = \frac{\partial \theta}{\partial u} \cdot \frac{\partial u}{\partial h_0} = \frac{-1}{L} \cdot \frac{1}{\sqrt{1 - ((h_1 - h_0)/L)^2}} \quad (5)$$

$$a_2 = \frac{\partial \theta}{\partial h_1} = \frac{\partial \theta}{\partial u} \cdot \frac{\partial u}{\partial h_1} = \frac{1}{L} \cdot \frac{1}{\sqrt{1 - ((h_1 - h_0)/L)^2}} \quad (6)$$

$$a_3 = \frac{\partial \theta}{\partial u} \cdot \frac{\partial u}{\partial L} = -\frac{(h_1 - h_0)}{L^2} \cdot \frac{1}{\sqrt{1 - ((h_1 - h_0)/L)^2}} \quad (7)$$

The standard uncertainty of length  $L$  on sine table ( $u_L$ ) may be classified as type B uncertainty source and a rectangular probability distribution is admitted. The uncertainty of height  $h_0$  on sine table ( $u_{h_0}$ ) may be classified as type B uncertainty source and a rectangular probability distribution is considered.

The standard uncertainty of gauge blocks length ( $u_{h_1}$ ) may be classified as type B uncertainty source having a normal probability distribution and it is determined using gauge block calibration results. Since the uncertainty of each individual gauge block ( $m$ ) may be expressed by  $u_m = A + B \cdot L_m$ , where  $A$  and  $B$  are constants and  $L_m$  is the length of an individual gauge block, the uncertainty of  $m$  gauge blocks stacked may be determined using Eq. (8).

$$u_{h_1} = \sqrt{m \cdot A^2 + A \cdot B \cdot L_m + B^2 \cdot \sum_{i=1}^m L_i^2} \quad (8)$$

The standard uncertainty of variability of measured angle was determined as type A uncertainty source having normal probability distribution, measuring three times the standard angle with the Electronic Level. The standard uncertainty of the Electronic Level resolution was determined as type B uncertainty source having rectangular probability distribution. The standard uncertainties of the temperature variation in relation to standard reference ( $u_{\Delta T_{20}}$ ), the temperature difference between standard and the instrument ( $u_{\Delta T_{diff}}$ ) and the roundness of the sine table cylinder ( $u_{\Delta C}$ ) were determined as type B uncertainty sources having rectangular probability distribution.

## 2.2 – Monte Carlo approach

Other approach used to determine measurement uncertainty is the Monte Carlo simulation, that is recommended when dealing with complex measurement processes in dimensional metrology [3, 4]. This method involves the determination of the probability distribution of the measurand by simulating the values of all variables involved in measurement. Fig. (2) shows a general scheme of the simulation procedure.

In the present investigation, the main function that deals with measurement process is represented by Eq. (1), in

which standard angle  $\theta_s$  is determined according to Eq. (3). In this expression, each variable have a characteristic distribution of its values that may be represented by a probability density function. Thereby, it is possible to simulate its values by generating random numbers according to the expected probability distribution and according to the specified range of variation. The theoretical background on this subject may be found in the literature [5].

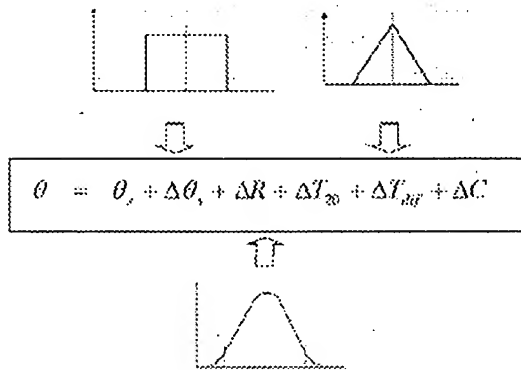


Figure 2 – General scheme of the simulation procedure

Random numbers were generated according to normal and rectangular distributions using Excel software and it was simulated 10000 trials admitting the probability distribution of each variable as pointed out to uncertainties determined by ISO GUM approach. The standard deviation of the measured angle was considered as the combined standard uncertainty and its value was compared with the values estimated using ISO GUM approach.

In these two approaches investigated, the expanded uncertainty was determined multiplying  $u_0$  by the coverage factor  $k=2$  adopting normal probability distribution with 95% probability.

### 3. RESULTS

The Electronic level was calibrated in 12 different angles on sine bar, starting at  $0^\circ$  and using  $0,5^\circ$  steps as angle interval to increase and decrease the angle.

The combined standard uncertainty was calculated according to ISO GUM to standard angle of  $306,9927$  min and the results are presented in Tab. (1). In this table, the standard uncertainties were determined considering: the circularity error of sine table cylinder as  $0,2 \mu\text{m}$ , the constants of the gauge blocks calibration as  $A=0,045$  and  $B=0,15$ , the experimental standard deviation of measured angle as  $0 \mu\text{m}$ , the resolution of the Electronic Level as  $1$  minute, the temperature variation during measurement of  $20 \pm 0,1^\circ\text{C}$ , the temperature difference of  $0,4^\circ\text{C}$ , the range of variation of the length  $h_0$  on sine table as  $12 \mu\text{m}$  and the range of variation of length  $L$  on sine table as  $16 \mu\text{m}$ .

As observed in Tab. (1), the resolution of the Electronic Level was the most significant effect influencing the

combined standard uncertainty of the Electronic Level. It was shown that expanded uncertainty is  $0,578$  min for a coverage probability of 95%.

The Results using Monte Carlo simulation method showed similar values of expanded uncertainty for the same coverage probability of 95% and it is  $0,586$  min.

Table 1 – Uncertainty of Electronic level – ISO GUM approach ( $\theta_s = 306.9927$  min)

Uncertainty source ( $u$ )	Symbol	Type	Prob. Distr.	DF	Sens. Coeff.	Results $u$
Sine bar height	$h_0$	B	Rectangular	$\infty$	$-4 \cdot 10^{-6}$	$1,7321 \mu\text{m}$
Gauge blocks height	$h_1$	B	Normal	$\infty$	$-4 \cdot 10^{-6}$	$0,1124 \mu\text{m}$
Sine bar Length	$L$	B	Rectangular	$\infty$	$4 \cdot 10^{-7}$	$2,3094 \mu\text{m}$
Variability of angle measured	$u_{\Delta\theta_s}$	A	Normal	2	1	$0,0000$ min
Electronic level resolution	$u_{\Delta R}$	B	Rectangular	$\infty$	1	$0,2886$ min
Temperature variation	$u_{\Delta T_{20}}$	B	Rectangular	$\infty$	1	$0,0000$ min
Differential temperature	$u_{\Delta T_{diff}}$	B	Rectangular	$\infty$	1	$0,0000$ min
Roundness	$u_{\Delta C}$	B	Rectangular	$\infty$	1	$0,0008$ min
Combined standard uncertainty						$u = 0,289$ min
Expanded uncertainty 95% $k=2$						$U = 0,578$ min

### 4. CONCLUSIONS

A comparison of ISO GUM approach and Monte Carlo simulation method was carried out using Electronic Level calibration results and it was observed good agreement between these techniques. Monte Carlo simulation reduced time spent in analysis and is suitable when elaborate mathematical expressions are developed to model the measurement.

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